The ϕ-dimension: A new homological measure

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Abstract. In [5], K. Igusa and G. Todorov introduced two functions ϕ and ψ, which are natural and important homological measures generalising the notion of the projective dimension. These Igusa-Todorov functions have become into a powerful tool to understand better the finitistic dimension conjecture.

In this paper, for an artin R-algebra A and the Igusa-Todorov function ϕ, we characterise the ϕ-dimension of A in terms either of the bi-functors $\mathrm{Ext}^i_A(-,-)$ or Tor's bi-functors $\mathrm{Tor}^A_i(-,-)$. Furthermore, by using the first characterisation of the ϕ-dimension, we show that the finiteness of the ϕ-dimension of an artin algebra is invariant under derived equivalences. As an application of this result, we generalise the classical Bongartz's result [**?**, Corollary 1] as follows: For an artin algebra A, a tilting A-module T and the endomorphism algebra $B = \mathrm{End}_A(T)^{op}$, we have that $\phi\dim(A) - \mathrm{pd}\,T \leq \phi\dim(B) \leq \phi\dim(A) + \mathrm{pd}\,T$.

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